Designs and the cone condition

Suppose there are 10 triangles on six points, so that each of the 15 pairs of points is covered by exactly two of the triangles. (Such a configuration actually exists, and is equivalent to a block design with $v = 6$, $k = 3$, $t = 2$, and $\lambda = 2$.) There can't be two disjoint triangles T_1 and T_2 in this collection, for the following reason. If there were, the remaining triangles would have to cover the 9 pairs crossing between T_1 and T_2, twice each. That's 18 crossing pairs to be covered. But each triangle yields at most two crossing pairs. So the other 8 triangles can't do the job and we have a contradiction.

On one hand, this is merely simple counting. However, viewed in greater generality, we made a structural conclusion about a certain block design using the convexity of pair coverage by triangles. My talk will further explore this technique, which I call the 'cone condition' for designs. It can be used to prove Fisher's inequality, bound block intersection numbers, and even completely kill certain hypothetical designs.

This is joint work with my doctoral supervisor Richard M. Wilson at the California Institute of Technology.