ECCC 2011

7th Annual
East Coast Combinatorics Conference

May 4–6, 2011

Acadia University, Wolfville, Nova Scotia

ACADIA
UNIVERSITY

AARMS

Wednesday, May 4
KC Irving Auditorium

1:15 - 1:25	Welcome
1:30 - 2:25	**Plenary speaker** P. Dukes: Designs and the cone condition
2:30 - 2:55	D. Pike: Colouring block designs
3:00 - 3:30	*Coffee, tea and cookies*
3:30 - 3:55	B. Hartnell: Graphs with exactly t different sizes of maximal independent sets of vertices
4:00 - 4:25	A. Finbow: On the packing chromatic number of some lattices
4:30 - 4:55	H. Chuangpishit: Nowhere-zero flows of graphs
5:00 - 5:25	A. Sanaei: New families of 3-existentially closed graphs

ACKNOWLEDGEMENTS. This workshop has been made possible through the support of the Atlantic Association for Research in the Mathematical Sciences (AARMS) and Acadia University.

Thursday, May 5
KC Irving Auditorium

9:00 - 9:55	**Plenary speaker** L. Stewart: Overlap representations and overlap numbers of graphs
10:00 - 10:30	*Coffee, tea, juice and muffins*
10:30 - 10:55	S. Fitzpatrick: Planar copwin critical graphs
11:00 - 11:25	S. Seager: Locating a robber on a graph
11:30 - 11:55	D. Dyer: Fast searching graphs with few searchers
12:00 - 12:25	A. Gagarin: Upper bounds for the bondage number of graphs on topological surfaces
12:30 - 2:00	*Conference Lunch: Garden Room, KC Irving Center*
2:00 - 2:25	F. Mendivil: De Bruijn sequences and IFS fractals
2:30 - 2:55	N. McKay: Wythoff sequences and partizan subtraction games
3:00 - 3:25	R. Keeping: Lessons in losing - an introduction to misère game theory
3:30 - 3:55	R. Dawson: Arithmetic polygons

Plenary Talks

Designs and the cone condition

Peter Dukes
University of Victoria

Suppose there are 10 triangles on six points, so that each of the 15 pairs of points is covered by exactly two of the triangles. (Such a configuration actually exists, and is equivalent to a block design with $v = 6$, $k = 3$, $t = 2$, and $\lambda = 2$.) There can't be two disjoint triangles T_1 and T_2 in this collection, for the following reason. If there were, the remaining triangles would have to cover the 9 pairs crossing between T_1 and T_2, twice each. That's 18 crossing pairs to be covered. But each triangle yields at most two crossing pairs. So the other 8 triangles can't do the job and we have a contradiction.

On one hand, this is merely simple counting. However, viewed in greater generality, we made a structural conclusion about a certain block design using the convexity of pair coverage by triangles. My talk will further explore this technique, which I call the 'cone condition' for designs. It can be used to prove Fisher's inequality, bound block intersection numbers, and even completely kill certain hypothetical designs.

This is joint work with my doctoral supervisor Richard M. Wilson at the California Institute of Technology.

Overlap representations and overlap numbers of graphs

Lorna Stewart
University of Alberta

An overlap representation of a graph is an assignment of sets to the vertices of the graph in such a way that two vertices are adjacent if and only if the sets assigned to them intersect and neither set is contained in the other. The overlap number of a graph is the minimum number of elements needed to form such a representation. We discuss overlap representations and overlap numbers in general and in cases where the graphs or the sets are required to have certain properties.

Contributed Talks

Nowhere-zero flows of graphs

Huda Chuangpishit
Dalhousie University

A graph G has a nowhere-zero k-flow if the edges of G can be oriented and assigned non-zero integers from the interval $(-k, k)$ such that for every vertex, the sum of the values on incoming edges equals the sum on the outgoing ones. This concept was introduced by Tutte in 1945. He showed that each planar graph is k-face colorable if and only if it has a nowhere-zero k-flow. There are three celebrated conjectures in this field, all due to Tutte. In sequel Jeager et al. introduced the concept of group connectivity of graphs as an extension of nowhere-zero flows. We will discuss the most important results on nowhere-zero flows and the concept of group connectivity.

Arithmetic polygons

Robert Dawson
St. Mary's University

We consider the question of the existence of equiangular polygons with edge lengths in arithmetic progression, and show that they do not exist when the number of sides is a power of two and do exist if it is any other even number. A few results for small odd numbers are given.

Fast searching graphs with few searchers

Danny Dyer
Memorial University

The edge search number of a graph is defined as the minimum number of cops needed to catch a fast invisible robber that may rest on a graph's vertices or edges. The fast search number is the minimum number of cops needed to capture such a robber in as few moves as possible. Paralleling the development of the edge search number, we will discuss graphs that require at most 3 cops to guarantee capture.

On the packing chromatic number of some lattices

Arthur Finbow
St. Mary's University

For a positive integer k, a k-packing in a graph G is a subset A of vertices such that the distance between any two distinct vertices from A is more than k. The packing chromatic number of G is the smallest integer m such that the vertex set of G can be partitioned as $V_1, V_2, \ldots, V_m$ where V_i is an i-packing for each i. It is proved that the planar triangular lattice T and the 3-dimensional integer lattice $\mathbb{Z}^3$ do not have finite packing chromatic numbers.

Planar copwin critical graphs

Shannon Fitzpatrick
University of Prince Edward Island

In the game of Cops and Robber, a cop tries to apprehend a robber as they move along edges of a graph. A Copwin Edge Critical graph, with respect to edge addition (deletion), is a graph that is not Copwin, but the addition (deletion) of any edge results in a Copwin graph. In this talk, I will discuss properties of Copwin Edge Critical graphs and give a characterization of those that are also planar.

Upper bounds for the bondage number of graphs on topological surfaces

Andrei Gagarin
Acadia University

The bondage number $b(G)$ of a graph G is the smallest number of edges of G whose removal results in a graph having the domination number larger than that of G. In a sense, the bondage number $b(G)$ measures integrity and reliability of the domination number $\gamma(G)$ with respect to the edge removal, which corresponds, e.g., to link failures in communication networks. We show that, for a graph G having the maximum vertex degree $\Delta(G)$ and embeddable on an orientable surface of genus h and a non-orientable surface of genus k,

$$b(G) \leq \min\{\Delta(G) + h + 2,\ \Delta(G) + k + 1\}.$$

This generalizes known upper bounds for planar and toroidal graphs. (Joint work with Vadim Zverovich, University of the West of England, Bristol, U.K.)

Graphs with exactly t different sizes of maximal independent sets of vertices

Bert Hartnell
St. Mary's University

We say that a graph G is in the collection M_t if there are precisely t different sizes of maximal independent sets of vertices in G. Thus the M_1 graphs are the well-covered ones (introduced by M. Plummer) where all the maximal independent sets are of one size. We examine graphs of higher girth belonging to M_t in the situation that the minimum degree is at least two. This is based on joint work with D. Rall.

Lessons in losing: an introduction to misère game theory

Rebecca Keeping
Dalhousie University

Combinatorial games are played by two players who alternate moves; there are no elements of chance, no hidden information, and no ties. Under *normal play* the last player to move wins the game, while under *misère play* the first player unable to move wins. Normal-play games have been extensively analyzed and exhibit 'nice' mathematical structure, including a notion of addition that forms the set of games into an Abelian group. Misère games have been much less studied, as almost all of the intuitive algebraic structure of combinatorial game theory seems to fall apart when we make the last player to move the loser. This talk will introduce combinatorial games in general before highlighting some of the challenges inherent in misère play, with examples from the game of *domineering*.

Wythoff sequences and partizan subtraction games

Neil A. McKay
Dalhousie University

Some of the most well-known combinatorial games go by the name Nim. In 1902, Bouton solved the standard form of Nim: from the given heaps of tokens, players alternate turns in which the remove some positive number of tokens from exactly one heap; the first player unable to move loses. Such a game is in the family of subtraction games (see Guy and Smith, 1956; and Fraenkel and Kotzig, 1987). In a subtraction game, the players may only remove n tokens from a heap if n is in the given subtraction set.

Whereas subtraction games as defined above are impartial games, subtraction games can be generalized to partizan games, in which the two players have different (finite) subtraction sets (Plambeck, 1995). In this talk, we present partizan subtraction where the subtraction sets are the elements the upper and lower Wythoff sequences. As these sequences are infinite, mutually disjoint, and non-periodic we see much different behaviour than in earlier investigations of subtraction games.

This work is joint with U. Larsson (Chalmers), R. J. Nowakowski (Dal), and A. A. Siegel (Dal).

De Bruijn sequences and IFS fractals

Franklin Mendivil
Acadia University

In this talk we show how de Bruijn sequences naturally arise in the context of IFS fractals. In particular, in modifying the “Chaos Game” to render the attractor of an IFS. The talk will introduce the relevant fractal topics.

Colouring block designs

David Pike
Memorial University

A block design with point set V and block set $\mathcal{B}$ is said to be c-colourable if the points of V can be partitioned into c sets called colour classes such that no block of $\mathcal{B}$ has all of its points in a single colour class. A design is said to be c-chromatic if it is c-colourable but not $(c-1)$-colourable. For all integers $c \geq 2$, $k \geq 6$ and $\lambda \geq 1$, we show that for sufficiently large v the obvious necessary conditions for the existence of a BIBD(v, k, λ) are sufficient for the existence of a c-chromatic BIBD(v, k, λ).

This is joint work with Daniel Horsley.

New families of 3-existentially closed graphs

Asiyeh Sanaei
Memorial University

A graph is n-existentially closed (n-e.c.) if for any two sets of vertices A and B with $|A| + |B| = n$ there exists a vertex $x \in V \setminus (A \cup B)$ that is adjacent to every vertex in A and to none in B. An operation is said to preserve the n-e.c. property if the result is an n-e.c. graph if applied on some n-e.c. graphs. Beside symmetric difference of two graphs that is shown to preserve the 3-e.c. property, in 2003 another 3-e.c. preserving graph operation was introduced. We have taken a different approach to the operation (denoted by $\bowtie$) that enables us to relax the requirement that both graphs considered be 3-e.c. We determine necessary and sufficient conditions that $G \bowtie H$ is 3-e.c. given that H is 3-e.c. The graph G can have as few as four vertices, which represents an improvement in comparison to when G is required to be 3-e.c.

This is a joint work with David A. Pike.

Locating a robber on a graph

Suzanne Seager
Mount Saint Vincent University

Consider the following game of a cop locating a robber on a connected graph. At each turn, the cop chooses a vertex of the graph to probe, and receives the distance from the probe to the robber. If she can uniquely locate the robber after this probe, she wins. Otherwise the robber may move to any vertex adjacent to his location other than the probe vertex. The cop's goal is to minimize the number of probes required to locate the robber, while the robber's goal is to avoid detection. This is a synthesis of the cop and robber game with the metric dimension problem. We consider some aspects of this game.